

## AP Calculus Bc Summer Review Packet

This packet includes material your teacher expects you to know when you begin the course. It is designed to be done over the course of the summer to provide practice and highlight the concepts you learned in AP Calculus AB.

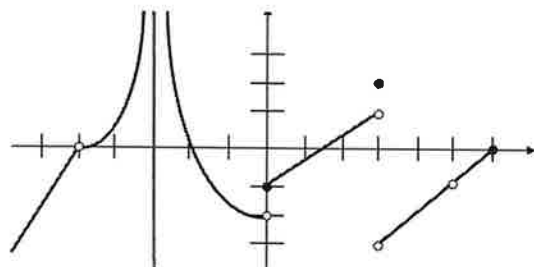
### Instructions:

- Complete the packet on loose leaf paper.
- Write your name and the course on the top of every sheet you use.
- Number your work, and do the problems in order.
- Copy each problem before showing your work.
- Check your answers as you go (answers are included at the end of the packet).

**The completed assignment is due on the first day of class and is worth 25 points.**

## Limits and Differentiation

1. Use the piecewise function graphed below to answer a-e that follow:



Evaluate the limits:

a.  $\lim_{x \rightarrow -5} f(x)$       b.  $\lim_{x \rightarrow -3} f(x)$       c.  $\lim_{x \rightarrow 0} f(x)$       d.  $\lim_{x \rightarrow 3} f(x)$

- e. Locate the x-values of any discontinuities (distinguish removable, non-removable).

2. Use the piecewise function given below to answer the questions a-f that follow:

$$f(x) = \begin{cases} 3 - x^2, & x \leq -1 \\ 4, & -1 < x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$$

Evaluate the limits:

a.  $\lim_{x \rightarrow 3} f(x)$       b.  $\lim_{x \rightarrow 0} f(x)$       c.  $\lim_{x \rightarrow 2} f(x)$       d.  $\lim_{x \rightarrow -1} f(x)$

- e. On what interval(s) is the function continuous?

- f. On what interval(s) is the function differentiable?

Evaluate each limit for #3-5:

3.  $\lim_{x \rightarrow -2} \frac{3x^2 + 21x + 30}{x^3 + 8}$

4.  $\lim_{x \rightarrow \infty} \frac{5x^3 - 2}{3 - e^{2x}}$

5.  $\lim_{x \rightarrow 6^-} \frac{|2x - 12|}{x - 6}$

6. Suppose  $f(x) = \begin{cases} \frac{x^2 - 3x}{x} & , x < 0 \\ 4x^2 + k & , x \geq 0 \end{cases}$ . For what value of  $k$  will  $f$  be continuous at  $x=0$ ?

Determine the derivative of each function for #7-#9.

7.  $f(x) = \sqrt{x-3} \sin x$

8.  $g(x) = \frac{x^2 + x - 1}{x^2 - 1}$

9.  $g(\theta) = \cos^2(3\theta^2 - 4)$

10. Determine the equation of the line tangent to the graph of  $f(x) = (x^2 - 1)^{2/3}$  at  $x=3$ .

11. Determine the  $x$  value(s) where  $f(x)$  from #10 has a vertical tangent line.

12. Determine open intervals where  $f(x)$  is increasing given  $f'(x) = \frac{x^2 + x - 6}{x^3 - 4x}$ .

### Integration and its applications

Evaluate each indefinite integral:

13.  $\int \sec^2(4x+1) dx$

14.  $\int y\sqrt{1+2y^2} dy$

15.  $\int \frac{dx}{x \ln x}$

Evaluate each definite integral:

16.  $\int_{-1}^2 x(1+x^3) dx$

17.  $\int_1^8 \frac{4}{x^2} dx$

18.  $\int_{-1}^1 \frac{dx}{1+x^2}$

19.  $\int_{-1}^1 (e^x - 1) dx$

20. Find the average value of the function  $f(x) = \sin x$  over the interval  $[0, \pi]$ .

21. Find the area enclosed by  $y = x^2$ ,  $y = \sqrt{x}$ ,  $x = \frac{1}{4}$ , and  $x = 1$ .

22. Find the area enclosed by  $x = y^2$  and  $y = x - 2$ .

23. Find the volume of the solid generated when the region enclosed by  $y = \sqrt{x}$ ,  $y = 2$ , and  $x = 0$  is revolved about the y-axis.

24. Find the volume of the solid generated when the region between the graphs of  $y = \frac{1}{2} + x^2$  and  $y = x$  over the interval  $[0, 2]$  is revolved about the x-axis.

25. Find the volume of the solid generated when the region in the first quadrant enclosed between the graphs of  $y = x^2$  and  $y = x$  is revolved about the y-axis.

**ANSWERS**

1. a. 0   b.  $\infty$    c. DNE  
d. DNE   e. removable:  $x=-5, 5$   
non-removable:  $x=-3, 0, 3$

2. a. 7   b. 4   c. 4  
d. DNE   e.  $(-\infty, -1) \cup (-1, \infty)$   
f.  $x = -1, x = 2$

3.  $\frac{3}{4}$

4. 0

5. -2

6.  $k = -3$

7.  $\sqrt{x-3} \cos x + \frac{\sin x}{2\sqrt{x-3}}$

8.  $\frac{-(x^2+1)}{(x^2-1)^2}$

9.  $-12\theta \cdot \cos(3\theta^2 - 4) \cdot \sin(3\theta^2 - 4)$

10.  $y - 4 = 2(x - 3)$

11.  $x = \pm 1$

12.  $(-3, -2) \cup (0, 2) \cup (2, \infty)$

13.  $\frac{1}{4} \tan(4x+1) + C$

14.  $\frac{1}{6}(1+2y^2)^{\frac{3}{2}} + C$

15.  $\ln|\ln x| + C$

16.  $\frac{81}{10}$

17.  $\frac{7}{2}$

18.  $\frac{\pi}{2}$

19.  $e - \frac{1}{e} - 2$

20.  $\frac{2}{\pi}$

21.  $\frac{49}{192}$

22.  $\frac{9}{2}$

23.  $\frac{32\pi}{5}$

24.  $\frac{69\pi}{10}$

25.  $\frac{\pi}{6}$

