

AP Calculus AB Summer Review Packet

This packet includes material your teacher expects you to know when you begin the course. It is designed to be done over the course of the summer to provide practice and highlight the concepts you learned in Honors PreCalculus.

Instructions:

- Complete the packet on loose leaf paper.
- Write your name and the course on the top of every sheet you use.
- Number your work, and do the problems in order.
- Copy each problem before showing your work.
- Check your answers as you go (answers are included at the end of the packet).

The completed assignment is due on the first day of class and is worth *25 points* for all math courses.

**AP Calculus AB Summer Review Packet
(Review of Honors PreCalculus)**

SHOW ALL WORK clearly.

Simplify as fully as possible. Leave final answers with only positive exponents.

1. $\frac{6x^{5/2} \cdot \sqrt[3]{x^2}}{12\sqrt{x}}$

2. $\frac{6x^2 - 12x^{3/2} + 8x^8}{2x^2}$

Simplify as fully as possible. Leave final answers in factored form.

3. $\frac{x^2 - 25}{x^2 - 4x - 5} \div \frac{2x + 10}{x^2 - 1}$

4. $\frac{(x-3)^3(x+2)^{-1}(x^2-9)}{(x-3)^5(x+2)^{-2}}$

Solve each equation for x over real numbers. Give exact answers.

5. $\sqrt{2x+7} - x = 2$

6. $2(x-2)^{3/2} = 54$

7. $e^{2x} - e^x = 0$

8. $4^{3x} = 2^{x^2+8}$

9. $2x^4 - 14x^2 + 24 = 0$

10. $x \ln x = x$

11. $|x^2 - 3x| = 6 - 4x$

12. $(x+1)^2(x-2) + (x+1)(x-2)^2 = 0$

13. $|4x-3| = 5\sqrt{x+4}$ (check #13 with calculator)

14. $\frac{x}{x-2} + \frac{1}{x+2} = \frac{-4}{x^2-4}$

15. Solve for x: $y^2 + 3xy - 4z = 8x$

16. Use the quadratic formula to find the *exact* solution to $x^4 - 5x^2 + 3 = 0$.

Inequalities

17. Solve and graph on a number line: $2x^3 - 7x^2 \geq -3x$ (use critical numbers and a sign graph)

18. Solve and graph on a number line: $\frac{x+3}{x-2} \geq 2$ (use critical numbers and a sign graph)

Factor Completely:

19. $4x^2(x+1)(x-3) - 8x(x-3)^2$

20. $2\sqrt{x} + 6x^{3/2} - 10x^2$ (Hint: Take out GCF)

Functions

21. Evaluate $\frac{f(x+h) - f(x)}{h}$ for $f(x) = x^2 - 2x$.

Use $f(x) = \sqrt{x-3}$ and $g(x) = x^2 - 1$ for #22-24.

22. Determine $(g \circ f)(x)$, in simplified form.

23. Determine $f^{-1}(g(-3))$.

24. State the domain of $(f \circ g)(x)$ in interval notation.

25. Sketch $f(x) = 4 - 2^{x-1}$ (no calculator), and state its domain and range.

26. Rewrite $f(x) = -2|x+3| - 4$ as a piecewise function.

27. Sketch $y = \sqrt{16 - x^2}$ (no calculator). State the domain and range.

28. Sketch the piecewise function neatly. Find the domain and range.

$$f(x) = \begin{cases} 3 - x^2, & x \leq -1 \\ 4, & -1 < x < 2 \\ 2e^{x-2} + 2, & x \geq 2 \end{cases}$$

29. Use the function in #28 to evaluate a-e:

- a. $f(3)$ b. $f(-3)$ c. $f(2)$ d. $f(0)$ e. $f(-1.3)$
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30. Discuss the end behavior of $g(x) = -4x^7 - 5x^5 + 3x^4 + 4x^3 - x^2 + 2x - 6$.

31. Determine the zeros, vertical asymptote(s), horizontal asymptote(s), and holes (as points) for

$$f(x) = \frac{x^2 + x - 6}{x^3 - 4x}$$

32. Use $f(x) = 2x^2 + 1$ to evaluate a) $f(x+h)$ and b) $f(x+h) - f(x)$.

33. Is $f(x) = 5x^4 - 3x^2 - 1$ odd, even, or neither? Justify analytically.

34. Describe any symmetry (about x-axis, y-axis, origin) for $y = \frac{x^2+1}{x}$.

35. Let $f(x) = \frac{3x+7}{x-2}$. Find a. $f^{-1}(x)$ and b. $f^{-1}(-10)$.

Equations, Graphs, and Intersections of graphs

36. Find the equation of the line in standard form that contains the vertex of the parabola $y = 2x^2 - 12x + 16$ and is parallel to $2x - 3y = 6$.

37. Find an equation for the parabola whose vertex is $(2, -5)$ and passes through $(4, 7)$. Express answer in standard quadratic form.

38. Find the point(s) of intersection of the curves $x^2 + 3x - y = 2$ and $y - 5x = 1$, analytically.

39. Graph to determine the point(s) of intersection of $x^2 + y^2 = 4$ and $x^2 + y^2 - 4x - 4y = -4$.

40. Find the domain of $f(x) = \frac{\sqrt{x+5}}{x+2}$. Express your answer in interval notation.

Trigonometry

41. Evaluate the exact value of each trigonometric expression (no calculators).

a. $\tan \frac{3\pi}{4}$

b. $\sec \left(-\frac{\pi}{2} \right)$

c. $\sin \left(-\frac{7\pi}{3} \right)$

d. $\cos(3\pi)$

42. Evaluate the exact value of each inverse trig function.

a. $\sin^{-1} \frac{\sqrt{3}}{2}$

b. $\arccos(-1)$

c. $\tan^{-1} \sqrt{3}$

d. $\cos^{-1}(0)$

43. Evaluate the exact value for the expression: $\sin(2\theta) + \cos^2\left(\frac{\theta}{2}\right)$, when $\theta = \frac{\pi}{3}$.

Solve for θ if $0 \leq \theta \leq 2\pi$.

44. $\cos\theta = -\frac{\sqrt{3}}{2}$

45. $\sin 2\theta - \cos\theta = 0$

46. $2\sin^2\theta - 3\sin\theta + 1 = 0$

47. $\cos^2\theta + \sin\theta = 1$.

ANSWERS to AP Calculus AB Summer Review Packet

1. $\frac{1}{2}x^{8/3}$
2. $3 - \frac{6}{\sqrt{x}} + 4x^6$
3. $\frac{x-1}{2}$
4. $\frac{(x+2)(x+3)}{x-3}$
5. $x=1$
6. $x=11$
7. $x=0$
8. $x=2, 4$
9. $x=\pm 2, \pm\sqrt{3}$
10. $x=e$; Note: $x=0$ is not part of domain
11. $x=-3, 1$
12. $x=-1, 2, \frac{1}{2}$
13. $x = \frac{49 \pm 5\sqrt{329}}{32}$; approx $-1.303, 4.365$
14. $x=-1$
15. $x = \frac{4z-y^2}{3y-8}$
16. $x = \pm \sqrt{\frac{5 \pm \sqrt{13}}{2}}$
17. $\left[0, \frac{1}{2}\right] \cup [3, \infty)$
18. $(2, 7]$
19. $4x(x-3)(x^2-x+6)$
20. $2\sqrt{x}(1+3x-5x^{3/2})$
21. $2x+h-2$
22. $(g \circ f)(x) = x-4$
23. 67
24. $(-\infty, -2] \cup [2, \infty)$
25. Domain = all real numbers,
Range = $(-\infty, 4)$
26. $\begin{cases} 2x+2, x < -3 \\ -2x-10, x \geq -3 \end{cases}$; domain = all reals,
range = $(-\infty, -4]$
27. Upper semicircle centered at $(0, 0)$ with radius $r=4$; domain is $[-4, 4]$, range is $[0, 4]$
28. Domain is all reals; Range $(-\infty, 2] \cup [4, \infty)$
29. a. $2e$ b. -6 c. 2 d. 4 e. 1.31
30. As $x \rightarrow -\infty, g(x) \rightarrow +\infty$, As $x \rightarrow +\infty, g(x) \rightarrow -\infty$
31. Hole at $(2, \frac{5}{8})$; vertical asymptote $x=0$ and $x=-2$; horizontal asymptote $y=0$
32. a) $f(x+h) = 2x^2 + 4xh + 2h^2 + 1$ and
b) $f(x+h) - f(x) = 4xh + 2h^2$
33. It is even because $f(-x) = f(x)$.
34. origin
35. $f^{-1}(x) = \frac{2x+7}{x-3}$; $f^{-1}(-10) = 1$
36. $2x - 3y = 12$
37. $y = 3x^2 - 12x + 7$
38. $(3, 16), (-1, -4)$
39. $(0, 2), (2, 0)$
40. $[-5, -2) \cup (-2, \infty)$
41. a. -1 b. undefined c. $-\frac{\sqrt{3}}{2}$ d. -1
42. a. $\frac{\pi}{3}$ b. π c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$
43. $\frac{\sqrt{3}}{2} + \frac{3}{4}$
44. $\frac{5\pi}{6}, \frac{7\pi}{6}$
45. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
46. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$
47. $0, \frac{\pi}{2}, \pi, 2\pi$