Calculus Summer Review Packet

The summer assignment for Calculus will help you to recall and reinforce some necessary Algebra and PreCalculus skills. In order to be successful in Calculus, these foundational skills must be strong.

Use your notebooks from prior courses and other available resources to help you. DO NOT simply submit the problems to an online problem solver. You will be required to solve similar problems in class and on tests.

You need to complete the entire packet. However, you will want to pace yourself. Do not try to complete it in one sitting – or leave it until the last few days of the summer.

Use loose leaf to complete the summer packet. Be sure that your work is neat, well-organized and complete. When you solve a problem, you are communicating your thought process. Be sure to communicate clearly.

DUE ON THE FIRST DAY OF CLASS

- Summer packet work completed on loose leaf.
  - Be sure to staple your pages!
- The summer work will be collected on the first day of class and represent your first grade in the course.
- A test on pre-requisite material will occur during the first week of class.

SIMPLIFYING EXPRESSIONS

Simplify as fully as possible. Leave final answers with only positive exponents.

1. $2(3x-1)^2-3(2x^2+x-1)$

2. $\left(\dfrac{a^5 b^{1/2} c^3 d^{-2}}{a b^{-1/2} c^5 d^2}\right)^{-2}$

3. $\dfrac{6x^{5/2}}{12\sqrt{x}}$

4. $\dfrac{6x^2-12x^{3/2}+8x^8}{2x^2}$

Leave final answer in factored form.

5. $\dfrac{x^2-25}{x^2-4x-5}\div\dfrac{2x+10}{x^2-1}$

6. $\dfrac{(x-3)^3(x+2)^{-1}}{(x-3)^5(x+2)^{-2}}$

SOLVING EQUATIONS

Solve each equation for x.

7. $3x^2-12=0$

8. $\left|x^2-3x\right|=-4x+6$

9. $\dfrac{x+1}{x}-\dfrac{x}{x+1}=0$

10. $\sqrt{2x+7}-x=2$

11. $2^{3x-1}-4=28$

12. $2x^4-14x^2+24=0$

13. $2(x-2)^{3/2}=54$

14. $3x^3-x^2=x$

15. $4x^3-8x^2-25x+50=0$

16. $\ln x=5$

17. $e^x=10$

18. $\log_4(x+3)=2$

19. $x^2e^{-x}-2xe^{-x}=0$

20. $\dfrac{3}{x-5}+\dfrac{2}{x+1}=\dfrac{6}{x^2-4x-5}$

SOLVING INEQUALITIES

Solve analytically and graph the solution on a number line. Give your solution in interval notation.

21. $-2\left|x-4\right|\le -14$.

22. $2x^3-7x^2\ge -3x$ (use critical numbers to make a sign graph)

23. $\dfrac{x+3}{x-2}\ge 2$ (use critical numbers to make a sign graph)

FACTORING

Factor Completely. Don't forget to factor out the GCF. ***Caution: Some of these are challenging!***

24. $4x^2(x+1)(x-3)-8x(x-3)^2$

25. $(x-3)^2-2x(x-3)$

26. $16x^4-1$

27. $2\sqrt{x}+6x^{3/2}-10x^2$

28. $(x^3)(4)(2x-5)^3(2)+(2x-5)^4(3x^2)$

29. $(1-x)(-1)(1+x^2)^{-2}(2x)+(1+x^2)^{-1}(-1)$ Challenge!

COMPOSITION OF FUNCTIONS

Given $t(x)=2x+5$ **and** $k(x)=3-x^2$

30. Determine $(k\circ t)(x)$, in simplified form.

31. Determine $t\big(k(x)\big)$, in simplified form.

Given $f(x)=-(x+3)^2+2$ $\quad g(x)=-\frac{1}{2}|x-5|$ $\quad h(x)=\sqrt{x+1}-3$

32. Evaluate $f(g(1))$.

33. Evaluate $g(h(3))$

GRAPHING

- Graph on graph paper. Do not use a calculator.
- State the domain and range
- Identify the transformations from the parent function

34. $f(x)=-(x+3)^2+2$

35. $h(x)=\sqrt{x+1}-3$

36. Rewrite $s(x)=-2|x+3|-4$ as a piecewise function. Then graph it. State the domain and range.

37. $y=\sqrt{16-x^2}$

38. Sketch the piecewise function below on graph paper, and then find the domain and range.

$$f(x)=\begin{cases}3-x^2, & x\le -1\\ 4, & -1<x<2\\ 3x-2, & x\ge 2\end{cases}$$

39. Use the function in #31 to evaluate a-d:

a. $f(3)$ b. $f(-3)$ c) $f(2)$ d. $f(0)$ e. $f(-1)$

ANALYZING FUNCTIONS

40. Discuss the end behavior of $g(x)=-4x^7-5x^5+3x^4+4x^3-x^2+2x-6$.

Determine the zeros and any vertical asymptote(s), horizontal asymptote(s), and holes (as ordered pairs):

41. $g(x)=\dfrac{x^2+x-6}{x^3-4x}$

42. $f(x)=\dfrac{1}{x^2-9}$

43. $f(x)=\dfrac{x^2-2}{x^2-x-2}$

44. $f(x)=\dfrac{x^3+1}{x^2-1}$

45. Use $f(x)=2x^2+1$ to evaluate a) $f(x+h)$ b) $f(x+h)-f(x)$ c) $\dfrac{f(x+h)-f(x)}{h}$

Symmetry: Even/Odd Functions – this is NOT referring to the degree of the function.

46. Is $f(x)=5x^4-3x^2-1$ odd, even, or neither? Justify analytically. Identify any symmetry.

47. Is $g(x)=2x^3-x$ odd, even or neither? Justify analytically. Identify any symmetry.

LINES AND INTERSECTIONS WITH LINES

48. Find the equation of the line, in standard form, that is perpendicular to the line containing (6,1) and (4,-3) and has the same y-intercept as 2x-3y = 6.

49. Find the equation of the line, in standard form, that goes through the vertex of the parabola $y = 2x^2 - 12x + 16$ and that is parallel to $2x - 3y = 6$.

50. Find the point of intersection of the lines $3x-y-7=0$ and $x+5y+3=0$ analytically (no calculator).

51. Find the point(s) of intersection of the curves $x^2+3x-y=2$ and $y-5x=1$, analytically.

TRIGONOMETRY

Evaluate each trigonometric expression. Use *exact* values (no calculators).

52. $\tan\dfrac{3\pi}{4}$

53. $\csc(-\dfrac{\pi}{6})$

54. $\sec\left(-\dfrac{\pi}{2}\right)$

55. $\cos\dfrac{7\pi}{4}$

56. $\sin\left(-\dfrac{7\pi}{3}\right)$

57. $\cos(3\pi)$

Evaluate each trigonometric expression for $\boldsymbol{\theta = \frac{\pi}{3}}$. Use *exact* values (no calculators).

58. $\sin(2\theta)$

59. $\cos^2(\frac{\vartheta}{2})$

Solve for θ if $0 \le \theta \le 2\pi$.

60. $\cos\theta = -\dfrac{\sqrt{3}}{2}$

61. $2\sin^2\theta - 3\sin\theta + 1 = 0$

Identify the domain and the range

62. $f(x)=\sin x$

63. $f(x)=\cos x+2$

64. $f(x)=\tan x$

MIXED REVIEW

65.

The graphs of f(x) and g(x) are given:

(a) State the values of f(-5) _____ and g(3) _____

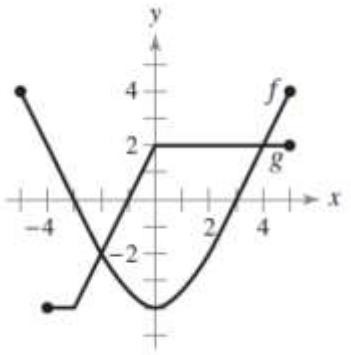
(b) for what values of x is f(x)=g(x)? _____

(c) Estimate f(-1) _____

(d) On what interval is f(x) decreasing? _____

(e) State the domain of f(x) _____

(f) State the range of f(x) _____



66. Simplify: $\dfrac{x^3-9x}{x^2-7x+12}$

67. Simplify: $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

68. Simplify: $\log_2 5+\log_2(x^2-1)-\log_2(x-1)$

69. Solve: $\log_2 x=3$

70. Solve: $\log_3 x^2=2\log_3 4-4\log_3 5$

71. Find the remainder when $x^5-4x^4+x^3-7x+1$ is divided by $x+2$.

72. Given $g(x)=2x^4-11x^3-x^2+30x$. Use your calculator to identify the following.

a. All roots

b. All local maxima

c. All local minima

d. g(-1), g(2), g(0), g(.125)

Answers to Calculus Summer Review Packet

1. $12x^2-15x+5$

2. $\dfrac{c^4d^8}{a^8b^2}$

3. $\dfrac{1}{2}x^2$

4. $3-\frac{6}{\sqrt{x}}+4x^6$

5. $\dfrac{x-1}{2}$

6. $\dfrac{x+2}{(x-3)^2}$

7. $x=\pm 2$

8. x = -3, 1

9. $x=-1/2$

10. $x=1$

11. $x=2$

12. $x=\pm 2,\pm\sqrt{3}$

13. $x=11$

14. $x=0,\dfrac{1\pm\sqrt{13}}{6}$

15. $x=-\frac{5}{2},2,\frac{5}{2}$

16. $x=e^5$

17. $x=\ln 10$

18. $x=13$

19. $x=0,2$

20. $x=13/5$

21. $x\le -3$ or $x\ge 11$, $(-\infty,-3]U[11,\infty)$

22. $\left[0,\dfrac{1}{2}\right]\cup[3,\infty)$

23. $(2,7]$

24. $4x(x-3)(x^2-x+6)$

25. $-(x-3)(x+3)$

26. $(2x-1)(2x+1)(4x^2+1)$

27. $2\sqrt{x}(1+3x-5x^{3/2})$

28. $x^2(2x-5)^3(14x-15)$

29. $(1+x^2)^{-2}(x^2-2x-1)$ or $\dfrac{x^2-2x-1}{(x^2+1)^2}$

30. $-4x^2-20x-22$

31. $11-2x^2$

32. 1

33. -3

34. parabola opening down, vertex (-3,2); D = all reals, R = $(-\infty, 2]$;shift left 3 units, reflect about x axis, shift up 2

35. domain = [-1, ∞) ; range = [-3, ∞); shift left 1, down 3

36. $\begin{cases} 2x+2, x<-3 \\ -2x-10, x\ge -3 \end{cases}$; domain = all reals, range = $(-\infty,-4]$

37. Top semicircle with center at (0, 0) and radius r = 4; domain is $[-4,4]$, range is [0, 4]

38. domain = all reals, range = $(-\infty, 2]$ ∪ $[4, \infty)$

39. a. 7 b. -6 c. 4 d. 4 e. 2

40. As $x\to-\infty, g(x)\to+\infty$, As $x\to+\infty, g(x)\to-\infty$

41. hole $\left(2,\frac{5}{8}\right)$; vert. asymp $x = 0$, $x = -2$; horiz. asymp $y = 0$, Zeros: (-3,0)

42. No holes; vert asymp: $x=\pm 3$; horiz asymp $y=0$; Zeros: none

43. No holes, vert asymp x = 2, x = -1, horiz asymp y=1; zeros $\left(\pm\sqrt{2},0\right)$

44. Hole $\left(-1,-3/2\right)$; vert asymp: x = 1; No horiz asymp; Zeros: none

45. a) $f(x+h) = 2x^2+4xh+2h^2+1$ and
b) $f(x+h)-f(x) = 4xh+2h^2$.

46. even , $f(-x) = f(x)$, symmetric w/ y axis

47. Odd, $g(-x) = -g(x)$, symmetric w/ origin

48. $x + 2y = -4$

49. $2x - 3y = 12$

50. $(2, -1)$

51. $(3, 16), (-1, -4)$

52. -1

53. -2

54. undefined

55. $\frac{\sqrt{2}}{2}$

56. $-\frac{\sqrt{3}}{2}$

57. -1

58. $\frac{\sqrt{3}}{2}$

59. 3/4

60. $\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

61. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

62. D: all reals, R: [-1,1]

63. D: all reals, R: [1,3]

64. D: $x \neq \pi/2 + n\pi$ R: all reals

65. a) 4 and 2 ; b) -2 and 4; c)-3.5, d)(-5,0); e) [-5, 5] f)[-4, 4]

66. $\frac{x(x+3)}{x+4}$

67. $\frac{5x}{5+x}$

68. $\log_2(5x+5)$

69. 8

70. $\pm\frac{4}{25}$

71. -89

72. a.) -1.5, 0, 2, 5
b) (1.067, 20.101)
c) (-0.890, -18.483), (3.948, -88.155)
d) g(-1)= -18, g(2)=0, g(0)=0, g(.125)=3.713