

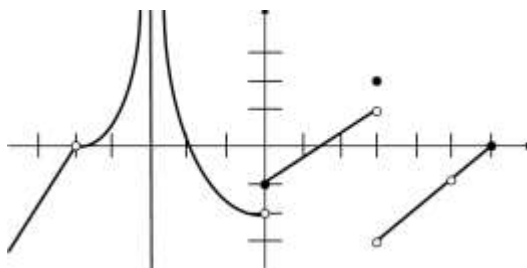
**AP Calculus BC Summer Review Packet
(Review of AP Calculus AB)**

The following problems represent a review of some of the fundamental concepts you need as you enter AP Calculus BC. They cover the basic skills you learned in AP Calculus AB. You are encouraged to refer to your notes, any textbooks, or online resources to guide you through this assignment. Answers are provided at the end of the document.

Complete the entire packet on loose leaf, showing all work and steps. All problems should be done without a calculator. Bring this work with you to the first class in September.

Limits and Differentiation

1. Use the piecewise function graphed below to answer a-e that follow:



Evaluate the limits:

a. $\lim_{x \rightarrow -5} f(x)$ b. $\lim_{x \rightarrow -3} f(x)$ c. $\lim_{x \rightarrow 0} f(x)$ d. $\lim_{x \rightarrow 3} f(x)$

- e. Locate the x-values of any discontinuities (distinguish removable, non-removable).

2. Use the piecewise function given below to answer the questions a-f that follow:

$$f(x) = \begin{cases} 3 - x^2, & x \leq -1 \\ 4, & -1 < x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$$

Evaluate the limits:

a. $\lim_{x \rightarrow 3} f(x)$ b. $\lim_{x \rightarrow 0} f(x)$ c. $\lim_{x \rightarrow 2} f(x)$ d. $\lim_{x \rightarrow -1} f(x)$

- e. On what interval(s) is the function continuous?

- f. On what interval(s) is the function differentiable?

Evaluate each limit for #3-5:

$$3. \lim_{x \rightarrow -2} \frac{3x^2 + 21x + 30}{x^3 + 8}$$

$$4. \lim_{x \rightarrow \infty} \frac{5x^3 - 2}{3 - e^{2x}}$$

$$5. \lim_{x \rightarrow 6^-} \frac{|2x - 12|}{x - 6}$$

$$6. \text{ Suppose } f(x) = \begin{cases} \frac{x^2 - 3x}{x}, & x < 0 \\ 4x^2 + k, & x \geq 0 \end{cases} . \text{ For what value of } k \text{ will } f \text{ be continuous at } x=0?$$

Determine the derivative of each function for #7-#9.

$$7. f(x) = \sqrt{x-3} \sin x$$

$$8. g(x) = \frac{x^2 + x - 1}{x^2 - 1}$$

$$9. g(\theta) = \cos^2(3\theta^2 - 4)$$

10. Determine the equation of the line tangent to the graph of $f(x) = (x^2 - 1)^{2/3}$ at $x=3$.

11. Determine the x value(s) where $f(x)$ from #10 has a vertical tangent line.

12. Determine open intervals where $f(x)$ is increasing given $f'(x) = \frac{x^2 + x - 6}{x^3 - 4x}$.

Integration and its applications

Evaluate each indefinite integral:

$$13. \int \sec^2(4x+1) dx$$

$$14. \int y \sqrt{1+2y^2} dy$$

$$15. \int \frac{dx}{x \ln x}$$

Evaluate each definite integral:

16. $\int_{-1}^2 x(1+x^3)dx$

17. $\int_1^8 \frac{4}{x^2} dx$

18. $\int_{-1}^1 \frac{dx}{1+x^2}$

19. $\int_{-1}^1 (e^x - 1)dx$

20. Find the average value of the function $f(x) = \sin x$ over the interval $[0, \pi]$.

21. Find the area enclosed by $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$, and $x = 1$.

22. Find the area enclosed by $x = y^2$ and $y = x - 2$.

23. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$, and $x = 0$ is revolved about the y-axis.

24. Find the volume of the solid generated when the region between the graphs of $y = \frac{1}{2} + x^2$ and $y = x$ over the interval $[0, 2]$ is revolved about the x-axis.

25. Find the volume of the solid generated when the region in the first quadrant enclosed between the graphs of $y = x^2$ and $y = x$ is revolved about the y-axis.

ANSWERS

1. a. 0 b. ∞ c. DNE
d. DNE e. removable: $x=-5, 5$
non-removable: $x=-3, 0, 3$

2. a. 7 b. 4 c. 4
d. DNE e. $(-\infty, -1) \cup (-1, \infty)$
f. $x = -1, x = 2$

3. $\frac{3}{4}$

4. 0

5. -2

6. $k = -3$

7. $\sqrt{x-3} \cos x + \frac{\sin x}{2\sqrt{x-3}}$

8. $\frac{-(x^2+1)}{(x^2-1)^2}$

9. $-12\theta \cdot \cos(3\theta^2 - 4) \cdot \sin(3\theta^2 - 4)$

10. $y - 4 = 2(x - 3)$

11. $x = \pm 1$

12. $(-3, -2) \cup (0, 2) \cup (2, \infty)$

13. $\frac{1}{4} \tan(4x+1) + C$

14. $\frac{1}{6} (1+2y^2)^{\frac{3}{2}} + C$

15. $\ln|\ln x| + C$

16. $\frac{81}{10}$

17. $\frac{7}{2}$

18. $\frac{\pi}{2}$

19. $e - \frac{1}{e} - 2$

20. $\frac{2}{\pi}$

21. $\frac{49}{192}$

22. $\frac{9}{2}$

23. $\frac{32\pi}{5}$

24. $\frac{69\pi}{10}$

25. $\frac{\pi}{6}$