AP Calculus AB Summer Review Packet (Review of Honors PreCalculus)

The following problems represent a review of some of the fundamental concepts you need as you enter AP Calculus AB. They cover material you have learned in past math classes, particularly Honors PreCalculus. You are encouraged to refer to your notes, any textbooks, or online resources to guide you through this assignment. Answers are provided at the end of the document. <u>Complete the entire packet on loose leaf, showing all work and steps. All problems should be done without a calculator. Bring this work with you to the first class in September. <u>Expect an assessment on the review material during the first week of classes.</u></u>

SHOW ALL WORK clearly.

Simplify as fully as possible. Leave final answers with only positive exponents.

1.
$$\frac{6x^{\frac{5}{2}} \cdot \sqrt[3]{x^2}}{12\sqrt{x}}$$
 2. $\frac{6x^2 - 12x^{\frac{3}{2}} + 8x^8}{2x^2}$

Simplify as fully as possible. Leave final answers in factored form.

3.
$$\frac{x^2 - 25}{x^2 - 4x - 5} \div \frac{2x + 10}{x^2 - 1}$$
 4. $\frac{(x - 3)^3 (x + 2)^{-1} (x^2 - 9)}{(x - 3)^5 (x + 2)^{-2}}$

Solve each equation for x over real numbers. Give exact answers.

- 5. $\sqrt{2x+7} x = 2$ 6. $2(x-2)^{\frac{3}{2}} = 54$ 7. $e^{2x} e^{x} = 0$
- 8. $4^{3x} = 2^{x^2+8}$ 9. $2x^4 - 14x^2 + 24 = 0$ 10. $x \ln x = x$

11.
$$|x^2 - 3x| = 6 - 4x$$

12. $(x + 1)^2(x - 2) + (x + 1)(x - 2)^2 = 0$

13.
$$|4x-3| = 5\sqrt{x+4}$$
 (check #13 with calculator) 14. $\frac{x}{x-2} + \frac{1}{x+2} = \frac{-4}{x^2-4}$

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15. Solve for *x*: $y^2 + 3xy - 4z = 8x$

16. Use the quadratic formula to find the *exact* solution to $x^4 - 5x^2 + 3 = 0$.

Inequalities

17. Solve and graph on a number line: $2x^3 - 7x^2 \ge -3x$ (use critical numbers and a sign graph)18. Solve and graph on a number line: $\frac{x+3}{x-2} \ge 2$ (use critical numbers and a sign graph)

Factor Completely:

19.
$$4x^{2}(x+1)(x-3)-8x(x-3)^{2}$$

20.
$$2\sqrt{x} + 6x^{\frac{3}{2}} - 10x^2$$
 (Hint: Take out GCF)

Functions

21. Evaluate
$$\frac{f(x+h)-f(x)}{h}$$
 for $f(x) = x^2 - 2x$.

Use $f(x) = \sqrt{x-3}$ and $g(x) = x^2 - 1$ for #22-24.

- 22. Determine $(g \circ f)(x)$, in simplified form.
- 23. Determine $f^{-1}(g(-3))$.

24. State the domain of $(f \circ g)(x)$ in interval notation.

25. Sketch $f(x) = 4 - 2^{x-1}$ (no calculator), and state its domain and range.

26. Rewrite f(x) = -2|x+3|-4 as a piecewise function.

27. Sketch $y = \sqrt{16 - x^2}$ (no calculator). State the domain and range.

28. Sketch the piecewise function neatly. Find the domain and range.

$$f(x) = \begin{cases} 3 - x^2, \ x \le -1 \\ 4, \ -1 < x < 2 \\ 2e^{x-2} + 2, \ x \ge 2 \end{cases}$$

29. Use the function in #28 to evaluate a-e:

a. f(3) b. f(-3) c) f(2) d. f(0) e. f(-1.3)

30. Discuss the end behavior of $g(x) = -4x^7 - 5x^5 + 3x^4 + 4x^3 - x^2 + 2x - 6$.

31. Determine the zeros, vertical asymptote(s), horizontal asymptote(s), and holes (as points) for

$$f(x) = \frac{x^2 + x - 6}{x^3 - 4x}.$$

32. Use $f(x) = 2x^2 + 1$ to evaluate a) f(x+h) and b) f(x+h) - f(x).

33. Is $f(x) = 5x^4 - 3x^2 - 1$ odd, even, or neither? Justify analytically.

34. Describe any symmetry (about x-axis, y-axis, origin) for $y = \frac{x^2+1}{x}$.

35. Let
$$f(x) = \frac{3x+7}{x-2}$$
. Find a. $f^{-1}(x)$ and b. $f^{-1}(-10)$.

Equations, Graphs, and Intersections of graphs

- 36. Find the equation of the line in standard form that contains the vertex of the parabola $y = 2x^2 12x + 16$ and is parallel to 2x 3y = 6.
- 37. Find an equation for the parabola whose vertex is (2,-5)and passes through (4, 7). Express answer in standard quadratic form.

38. Find the point(s) of intersection of the curves $x^2 + 3x - y = 2$ and y - 5x = 1, analytically.

39. Graph to determine the point(s) of intersection of $x^2 + y^2 = 4$ and $x^2 + y^2 - 4x - 4y = -4$.

40. Find the domain of $f(x) = \frac{\sqrt{x+5}}{x+2}$. Express your answer in interval notation.

Trigonometry

41. Evaluate the exact value of each trigonometric expression (no calculators).

a.
$$tan\frac{3\pi}{4}$$
 b. $sec\left(-\frac{\pi}{2}\right)$ c. $sin\left(-\frac{7\pi}{3}\right)$ d. $cos(3\pi)$

42. Evaluate the exact value of each inverse trig function.

a.
$$Sin^{-1}\frac{\sqrt{3}}{2}$$
 b. $arccos(-1)$ c. $Tan^{-1}\sqrt{3}$ d. $Cos^{-1}(0)$

43. Evaluate the exact value for the expression: $sin(2\theta) + cos^2(\frac{\theta}{2})$, when $\theta = \frac{\pi}{3}$.

Solve for θ if $0 \le \theta \le 2\pi$.

44.
$$\cos\theta = -\frac{\sqrt{3}}{2}$$

45. $\sin 2\theta - \cos \theta = 0$
46. $2\sin^2\theta - 3\sin\theta + 1 = 0$
47. $\cos^2\theta + \sin\theta = 1$.

1.
$$\frac{1}{2}x^{\frac{N}{3}}$$

2. $3 - \frac{6}{\sqrt{x}} + 4x^{6}$
3. $\frac{x-1}{2}$
4. $\frac{(x+2)(x+3)}{x-3}$
5. $x=1$
6. $x=11$
7. $x=0$
8. $x=2,4$
9. $x=\pm 2,\pm\sqrt{3}$
10. $x=e$; Note: $x=0$ is not part of domain
11. $x=-3,1$
12. $x=-1,2,\frac{1}{2}$
13. $x=\frac{49\pm5\sqrt{329}}{32}$; approx -1.303, 4.365
14. $x=-1$
15. $x=\frac{4z-y^{2}}{3y-8}$
16. $x=\pm\sqrt{\frac{5\pm\sqrt{13}}{2}}$
17. $\left[0,\frac{1}{2}\right] \cup [3,\infty)$
18. $(2,7]$
19. $4x(x-3)(x^{2}-x+6)$
20. $2\sqrt{x}(1+3x-5x^{\frac{N}{2}})$
21. $2x+h-2$
22. $(g \circ f)(x)=x-4$
23. 67
24. $(-\infty,-2] \cup [2,\infty)$
25. Domain = all real numbers,
Range = $(-\infty,4)$
26. $\left\{-\frac{2x+2,x<-3}{-2x-10,x\geq-3};$ domain = all reals,
range = $(-\infty,-4]$

- 27. Upper semicircle centered at (0, 0) with radius r = 4; domain is [-4,4], range is [0, 4]
- 28. Domain is all reals; Range $(-\infty, 2] \cup [4, \infty)$
- 29. a. 2e b. -6 c. 2 d. 4 e. 1.31
- 30. As $x \to -\infty$, $g(x) \to +\infty$, As $x \to +\infty$, $g(x) \to -\infty$
- 31. Hole at $\left(2, \frac{5}{8}\right)$; vertical asymptote x = 0 and x = -2; horizontal asymptote y = 0
- 32. a) $f(x+h) = 2x^2 + 4xh + 2h^2 + 1$ and b) $f(x+h) - f(x) = 4xh + 2h^2$
- 33. It is even because f(-x) = f(x).
- 34. origin

35.
$$f^{-1}(x) = \frac{2x+7}{x-3}; f^{-1}(-10) = 1$$

36. $2x - 3y = 12$
37. $y = 3x^2 - 12x + 7$
38. $(3, 16), (-1, -4)$
39. $(0,2), (2,0)$
40. $[-5, -2) \cup (-2, \infty)$
41. a. -1 b. undefined c. $-\frac{\sqrt{3}}{2}$ d. -1
42. a. $\frac{\pi}{3}$ b. π c. $\frac{\pi}{3}$ d. $\frac{\pi}{2}$
43. $\frac{\sqrt{3}}{2} + \frac{3}{4}$
44. $\frac{5\pi}{6}, \frac{7\pi}{6}$
45. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$
46. $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$
47. $0, \frac{\pi}{2}, \pi, 2\pi$